Metrics for Application of Revenue Sensitivity Analysis to Predict Market Power Coalitions in Electricity Markets

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Abstract-- This paper explores a mathematical method for detecting groups of generators in an electric power system that have the potential to benefit from exercising market power. Applications of this method include metrics for measuring or detecting the possibility of market power. This paper focuses on the properties of revenue and dispatch to bid sensitivity matrices, and develops methods of identifying load pockets from the sensitivity matrices, and how the matrices can provide metrics for market power.

***Index Terms*—Optimal power flow, electricity markets, market power**

I. Introduction

PHYSICAL constraints in the electrical transmission network limit the sources of generation for some groups of load, or load pockets. Generators inside load pockets often have opportunities to exercise market power by submitting high bids. Groups of generators may find ways to adjust their outputs in order to constrain a transmission line, creating a load pocket from which they can profit. This paper explores a mathematical method for detecting groups of generators in an electric power system that have the potential to benefit from exercising market power due to a load pocket. Applications of this method include metrics for measuring or detecting the possibility of market power. This method could be used by FERC or an ISO in market power mitigation, or by generators or power marketers in determining their strategies.

Harvey and Hogan [1], [2] discuss the need for narrowing the focus of market power detection, since simulation studies alone cannot replicate all of the real-world constraints of the market. They discuss several examples where prices may be above the competitive levels determined by simulation studies, but market power is not being exercised. Alvarado and Rajaraman [3] discuss conduct tests to determine whether generators' strategies are compatible with those of a price taker.

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Glavitsch and Alvarado [4] explore solving two optimal power flows (OPF) to determine price signals in order to relieve congestion at minimum cost. They calculate additive price signals, and when these are applied in the second OPF ignoring all constraints, the two OPFs have the same solution for power generated/consumed at each node, but different solutions for nodal prices. Lesieutre, Thomas and Mount [5] develop a method to calculate matrices of dispatch to bid and revenue to bid sensitivities for the generators in a power system. The matrices are also calculated using two OPFs. The first OPF establishes an operating point using all market information. For the second OPF, block offers are replaced by the nodal prices from the first OPF, and generator production limits are relaxed. This allows for incremental analysis to identify generators that can increase their offer prices, and hence revenues, without affecting dispatch. Generators with this ability are likely to be located in a load pocket and have the potential to benefit from exercise of market power.

Similar work is presented by He and Song [6], who use a two-level optimization problem with ac power flow equations to solve for Nash equilibrium including coalitions. This algorithm allows generators or system operators to determine which coalitions are the most beneficial to the participants. One limitation to the work is the combinatorial explosion that occurs with larger systems.

The outline of this paper is as follows: section II focuses on the properties of revenue and dispatch to bid sensitivity matrices, sections III and IV develop methods of identifying load pockets from the sensitivity matrices, and section V discusses conclusions and future work.

II. Properties of Revenue and Dispatch to Bid Sensitivity Matrices

As a method to identify load pockets and generators with the ability to exercise market power, Lesieutre, Thomas and Mount [5] compute two sensitivity matrices in their work: dispatch-to-offer and revenue-to-offer. These matrices are calculated using two optimal power flows. The first OPF uses all market information to establish an operating point. Block offers are replaced with nodal prices from the first OPF, and generator production limits are relaxed in the second OPF. Through incremental analysis with the two OPFs, the dispatch-to-offer and revenue-to-offer sensitivity matrices are calculated. These matrices are used to identify load pockets, and also to identify generators that can exercise market power through their location in or near a load pocket. In this paper, we extend this analysis.

The dispatch-to-offer sensitivity matrix M has the following properties: it is symmetric, has all negative diagonal entries, it has all negative real eigenvalues, and has at least one zero eigenvalue. The revenue-to-offer sensitivity matrix A is nearly symmetric, all of its diagonal entries are negative, its eigenvalues are real, and the sum of each row is nonnegative.

In obtaining the sensitivity matrices in [5], an incremental model was developed by linearizing about the solution to an optimal power flow. This model assumes that in the region of the OPF solution, dispatch and revenue can be approximated as linear functions of generator bids. Thus, the revenue-offer sensitivity matrix is only valid in the region close to the OPF solution. As generators change bids, the OPF solution will change, *as will the binding constraints*. Consequently, a load pocket identified through the sensitivity matrices may exist only under specific operating conditions.

The two OPFs can be run with quadratic or piecewise-linear bids. Future work on this topic includes extending the analysis to multi-part bids with startup and shutdown costs, and including bids for reserves and other ancillary services.

The symmetry and near-symmetry of the matrices demonstrate that changes in dispatch due to changes in offers of two generators are reciprocal. For example, in a two-bus, two-generator system, if generator A increases its offer relative to the other generator, its dispatch will decrease and the dispatch of generator B will increase. The opposite will happen if generator B increases its offer. Since the two generators are connected by the same transmission line, the changes in dispatch will be similar in magnitude for comparable changes in offer.

The negative diagonal elements of the revenue – bid sensitivity matrix indicate that no generator by itself can increase its revenue by raising its bid. According to economic theory, for a perfectly competitive market, this means that all generators are initially submitting bids at marginal cost. Generators that are perfect competitors are price-takers, and changing their bids will not impact prices. Electricity markets do not exhibit perfect competition, and the pivotal generator(s) that set the price can have an impact on this price. A price taker will be either fully dispatched or not dispatched depending on its bid, and the price it receives will be based on the price set by the marginal, or pivotal, generator in its area.

We assume that dispatch decreases linearly with increasing bid; this is true for an unconstrained price taking generator. Fig. 1 shows the relationships of dispatch, revenue, and profit to bid for a non-price taking generator. The revenue and profit curves are based on the assumption that this is paid what it bids. Generators bidding at marginal cost are operating at the maximum of this parabolic curve, as shown in Fig. 1. These curves are the same that would be obtained using quadratic bid curves and without any binding constraints.

Studying generator strategies in terms of revenue implies that the generators have zero marginal costs, such as hydro generators, or that marginal costs are constant for all output levels and independent of bid. Rational market participants should optimize profit, not revenue. Costs may or may not be coupled to bids, and will differ with fuel type. The diamond points on the revenue and profit curves in Fig. 1 indicate the maxima. The third curve shows profit when bids are equal to marginal costs, while the fourth curve shows profit when marginal costs are constant and decoupled from bids. In this case, the profit and revenue maxima occur at the same point, at a bid of \$30/MW. When bids equal marginal costs, the profit maximum occurs at a slightly lower bid, in this case at \$25/MW. The fifth curve shows profit when marginal costs are declining and bid is decoupled from marginal cost. The maximum profit point now depends on how much marginal costs decline. Many thermal generators have declining marginal costs for part of their operating range, since the generators reach maximum efficiency at high power outputs.

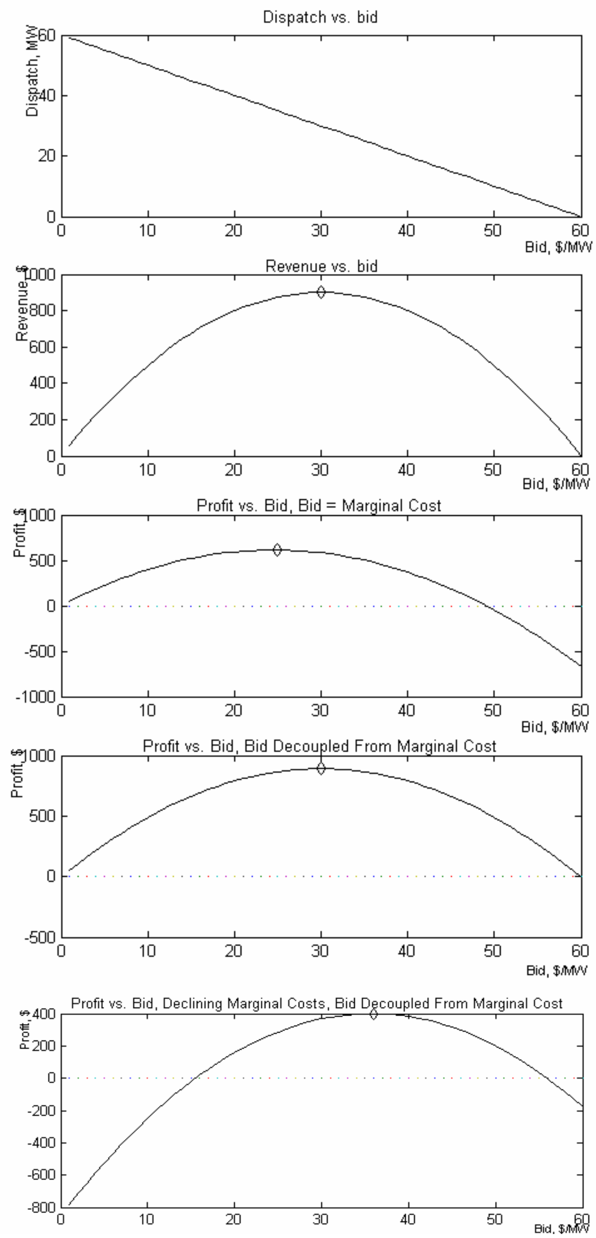

Fig. 1: Relationships of dispatch, revenue and profit to bid for quadratic costs for a single generator in an unconstrained system

The zero eigenvalue of the dispatch-to-offer sensitivity matrix M is associated with an eigenvector of all ones. This occurs because if all generators increase their offers in the same proportion, there will be no change in dispatch[1]. There is not a zero eigenvalue in the revenue-to-offer sensitivity matrix A because if all generators increase their offers proportionately, all will increase their revenues since marginal prices will increase.

The dispatch-to-offer sensitivity matrix M is always symmetric, for both quadratic and piecewise linear bid curves. Since it is symmetric, it has distinct real eigenvalues and orthogonal eigenvectors.

The row sums in the revenue-to-offer sensitivity matrix are always nonnegative because if all generators increase their offers in the same proportion, the dispatch will remain the same and all generators will increase their revenues. Likewise, the sum of the row sums is nonnegative. According to [5], the dispatch-to-offer matrix tends to have row sums and sum of the row sums close to zero. In practice this is not always true. It tends to be true for cases that have a clearly identifiable load pocket, such as the IEEE 30-bus test system, but not for other systems.

Looking at row sums or column sums in the dispatch-offer sensitivity matrix are the same due to the symmetry of the matrix.

The dispatch-to-offer matrix usually has all zero and negative eigenvalues. The revenue-to-offer matrix has a mix of negative and positive eigenvalues. The dispatch-to-offer matrix, M, and the revenue-to-offer matrix, A, have the relationship shown in (1), where z is dispatch and λ the locational marginal price and diag(x) is a matrix with the values of the vector x along its diagonal.

$$A = diag(z) + M*diag(\lambda) \tag{1}$$

We know the following about sums of square matrices from linear algebra theory [9]:

1. trace(A+B) = trace(A) + trace(B), where trace is the sum of diagonal elements
2. $\Sigma(\Lambda(A+B)) = \Sigma(\Lambda(A)) + \Sigma(\Lambda(B))$, where Λ represents the list of eigenvalues of the matrix.

In normal operation, generators are dispatched to produce zero or positive amounts of power, so diag(z) will have all nonnegative eigenvalues. The locational marginal prices will usually be nonnegative, but this is not always the case. Thus $\Sigma(\Lambda(A)) = \Sigma(\Lambda(z)) + \Sigma(\Lambda(M*diag(\lambda)))$. This is why the revenue-to-offer sensitivity matrix often has more positive eigenvalues than the dispatch matrix.

Off-diagonal elements can take any value. They represent the sensitivity of one generator's revenues to the change in bid of another generator. Off-diagonal elements in a row represent the sensitivity of a particular generator's revenue to changes in bid from the other generators. Off-diagonal elements in a column indicate the sensitivity of each generator to a change in bid of one generator. In theory, many of the off-diagonal sensitivities should be zero for an infinitesimal step. However, due to the finite step size used in incremental analysis and system constraints, these values are nonzero.

[1] This is, of course, only true if there is no demand-side response to prices.

III. Matrix Sums as a Metric for Market Power

Given the matrix of revenue to bid sensitivities discussed above, what is the best way to identify groups of generators with the potential to exercise market power? We examine three possibilities: positive-sum row subsets, positive-sum column subsets, and positive-sum submatrices. In all three cases, there is always at least one positive-sum subset – the subset of the whole row, column, or matrix. We are assuming that demand is fixed and inelastic. Thus when all generators raise their bids proportionally, demand will not respond, and all generators will see increased revenues. Demand bidding could be represented by additional generators with modified cost/bid curves.

There are two tests for the potential of market power for a group of generators from the revenue – bid sensitivity matrix:

1. Does raising the bids of all generators in the group benefit each generator in the group? (Row sums)
2. Does raising the bid of one generator in the group benefit all other generators in the group? (Column sums)

The first criterion is the more obvious indicator of collusion, while the subtler second criterion is necessary for the collusion to continue. If a row has a positive sum, then raising the bid of the generators in the group benefits the generator corresponding to that row. However, it does not necessarily benefit all generators in the group, and to continue cooperating in raising prices a system of side payments or explicit collusion may be necessary. If a column has a positive sum, raising the bid of the generator corresponding to that column benefits all generators in the group. For successful implicit collusion, all rows and columns of the group of generators must have positive sums. One easy way to verify this is to compute the sum of the entire submatrix. If it is positive, the group meets both tests for possible stable collusion[2].

We develop two Matlab programs to test randomly generated matrices for these conditions. Both programs use a combinatorial function to find all one-bit departures from a given binary vector. This vector indicates which rows and/or columns are members of the subset being checked for a positive sum. The randomly generated matrices have negative diagonal elements, indicating that no generator can act alone to increase its revenue by increasing its bid. The sum of all elements in the matrix is positive, since we are assuming inelastic demand, so that if all generators raise their bids, all will increase their revenues.

Examining submatrix sums allows us to check for both implicit and explicit collusion. Generators who can together raise bids and profit can learn this through repeated bidding, without communicating with each other, colluding implicitly. These same generators could of course discuss their strategies and explicitly collude, an illegal practice in most markets. The submatrix sums might also identify groups of generators using side payments to explicitly collude. For example, a group observed repeatedly following a certain bidding pattern that appears unprofitable for some group members may

[2] An interesting area of future research is stability analysis of the collusive point over multiple periods.

indicate that these members are being compensated outside of the standard market.

A. Row Sums

Studying row sums alone is one way to identify potential load pockets. However, many positive row sum sets do not meet both of the criteria for market power. For example, the 5 x 5 matrix shown in Table I has a positive- sum subset in row 4 of elements 3 and 4:

Yet, as Table II shows, the same group of generators does not have a positive sum in rows 1 or 5. This indicates that the net change in revenue for these three generators increasing bids is only positive for generator three. Generators one and five are better off leaving their bids at marginal cost than raising them with generator three, and thus are not likely to collude with generator three to raise prices. The only way that generators one and five would cooperate with generator three is if they are somehow reimbursed, such as with side payments or if one company owns all three generators. However, generator three's gain is not enough to offset the losses of generators one and five.

Table I: Randomly generated 5 x 5 matrix[3]

-18	-25	30	-55	-2
-29	-26	62	2	-2
30	51	-36	-36	-1
54	-1	36	-24	10
52	7	18	3	-61

Table II: Subset of 5 x 5 matrix

					Row Sum
-18	-25	30	-55	-2	
-29	-26	62	2	-2	
30	51	**-36**	**-36**	-1	-72
54	-1	**36**	**-24**	10	12
52	7	18	3	-61	
			Submatrix Sum		-60

B. Column Sums

Column sums have the same problem as row sums – they identify only one of the two necessary conditions for potential to exercise market power. Consider a simple two generator system, with revenue to bid sensitivities shown in Table III. When generator one increases its bid, it loses, and the group also loses, as seen in the negative sum for the first column. When generator two increases its bid, it loses, but the group gains, shown by the positive second column sum. In this case, generator two benefits from generator one or both generators increasing bids, but generator one does not benefit in either case. The submatrix sum, which is the sum of the row sums or the sum of the column sums, is negative, since the entire group does not benefit when both generators raise their bids.

Table III: Revenue to bid sensitivities for a 2 generator system

			Row Sums
	-8	6	-2
	2	-1	1
Column Sums	-6	5	Submatrix Sum = -1

Table IV shows an example for a larger system, where the column sums program found a positive sum for column 4 using columns 4 and 5. However, column 5 has a negative sum, as seen in Table IV. This indicates that raising the bids of generators 4 and 5 does not benefit both generators. As with the row sums, a system of explicit collusion such as side payments or joint ownership would be necessary for this group to continue increasing prices together. The submatrix sum, which indicates the overall benefit to the group, is negative, demonstrating that this is not a likely situation for market power.

Table IV: Randomly generated 5 x 5 matrix

-16	2	-9	76	-16
22	-34	4	100	-3
-45	2	-20	99	-18
57	12	15	-24	-27
11	1	3	24	-21

Table V: Subset of 5 x 5 matrix

	-16	2	-9	76	-16	
	22	-34	4	100	-3	
	-45	2	-20	99	-18	
	57	12	15	**-24**	**-27**	
	11	1	3	**26**	**-21**	Submatrix Sum
Column Sums				2	-48	-46

When the column sums for a group are positive, collusion can take place implicitly, without any exchange of money between the group members. In the above example, if generator 4 increases its bid, all generators in the group benefit, but generator 4 loses. Alternatively, if all four generators in the group raise their bids, generator 4 benefits (positive sum of row 4), but the other generators do not (negative row sums for rows 1, 3, and 5). When all members of the group have positive column sums, they all benefit from the others raising their bids.

C. Submatrix Sums

A positive-sum submatrix represents a group of generators for which the rows and columns associated with the group have a positive sum. For a submatrix to have a positive sum, the off-diagonal elements must be positive and of greater magnitude than the diagonal elements. Since submatrix sums combine row and column sums, they take into account both conditions for market power potential. If submatrix sums can be efficiently computed, they are a potential metric for market power. High submatrix sums indicate a strong incentive to exercise market power.

D. Improving Performance of Submatrix Search Program

There are some methods that can speed up the submatrix search program. First, we can limit the searching to groups

[3] The randomly generated matrices shown here used to demonstrate the general ideas of submatrix sums, but are not actual power system sensitivity matrices.

under some maximum size. Second, we can prune the search space at each step.

Costs of collusion increase with the number of colluders in a group, and for large groups, the incentives to collude tend to be less than the benefits of participating competitively in the market [10]. Legally coordinating actions among group members becomes impossible for groups with more than a few members. One exception to this is a large group of generators jointly owned by one company. We limited the maximum group size in the search to 2, 3, 4, and the entire group of generators. Table VI shows the computation times for 100 trials with randomly generated 20 x 20 matrices. Limiting the group size to two generators halves the computation time of searching for groups of three or more. The savings of limiting the group size to three are not significantly greater than searching with unlimited group size.

Table VI: Computation times for different group sizes, 20 x 20 matrices

Maximum Group Size	Mean Computation Time	Minimum Computation Time	Maximum Computation Time
2	$6.7593*10^{-7}$	$5.3237*10^{-7}$	$1.9908*10^{-6}$
3	$1.3528*10^{-6}$	$1.0764*10^{-6}$	$2.0784*10^{-6}$
4	$1.5068*10^{-6}$	$1.0764*10^{-6}$	$2.5347*10^{-6}$
10	$1.5768*10^{-6}$	$1.0764*10^{-6}$	$2.3495*10^{-6}$
20	$1.485*10^{-6}$	$1.0764*10^{-6}$	$2.3495*10^{-6}$

We pruned the combinations of generators at each step so that the program looks only at the B*n largest sums from the previous step. B is an integer, and n is the dimension of the matrix. This limits the number of new 1-bit different combinations generated by the combinatorial function. We began with B as 10, and tried different values of B for the same matrices. For 8x8 and 20x20 matrices, the program finds the same positive-sum subset for values of B three and greater, with the lowest values having the shortest computation times. For 100x100 matrices, B as low as two produces the same results as higher levels, but in less time. The computation time decreases quadratically with decreasing B.

The running times of both programs increase with matrix size. Both currently stop when the first positive-sum subset is found, so the submatrix program takes longer since it finds positive-sum subsets less often, and when it fails, must run through the entire matrix. The program works for matrices up to 200 x 200, but takes a long time to run for larger matrices.

IV. Eigenvalue Methods to Locate Load Pockets

The Lesieutre, Thomas and Mount paper describes a spectral analysis method that could be developed to identify load pockets [5]. In this method, the eigenvalues of the revenue-offer sensitivity matrix represent gains. Any change in nodal prices in the system can be represented as a sum of scaled eigenvectors of the revenue-offer sensitivity matrix, and the corresponding change in dispatch will be the change in price scaled by the associated eigenvalues. Small eigenvalues indicate presence of a load pocket, and large values in the corresponding eigenvector show the generators that are part of the load pocket.

A method based on eigenvalues can be applied to detect many cases of market power, not just a group of generators raising bids together. In some situations, market power is exercised by two generators raising their bids in different proportions; in this way, the high bid increases the LMP paid to both generators, but one generator may keep its bid slightly lower in order to have more of its power dispatched. Another interesting case of market power is when one generator lowers its bid to induce congestion while a second generator submits a high bid. For this case to be profitable to the two generators, the LMP paid to the generators would have to be strongly correlated to the bid of the second generator and very weakly correlated to the first generator's bid. For the second generator to influence the LMP, its bid needs to be low enough that some of its power is dispatched. If this is the case, the first generator will be fully dispatched, and will receive the LMP influenced by the second generator's high bid.

These observations about eigenvalues can be applied to search for positive-sum subsets in less time than the combinatorial method, as shown in Fig. 2. Instead of going through all combinations of submatrices, we begin by computing the eigenvalues and eigenvectors of the revenue-offer sensitivity matrix. The search starts with the eigenvector corresponding to the smallest eigenvalues. The first groups of generators to test are those with the largest value in this eigenvector, then one-bit departures from these combinations, the other generators corresponding to large values in the eigenvector, then the eigenvector with the next-smallest eigenvalue, and so on until a positive-sum submatrix is found.

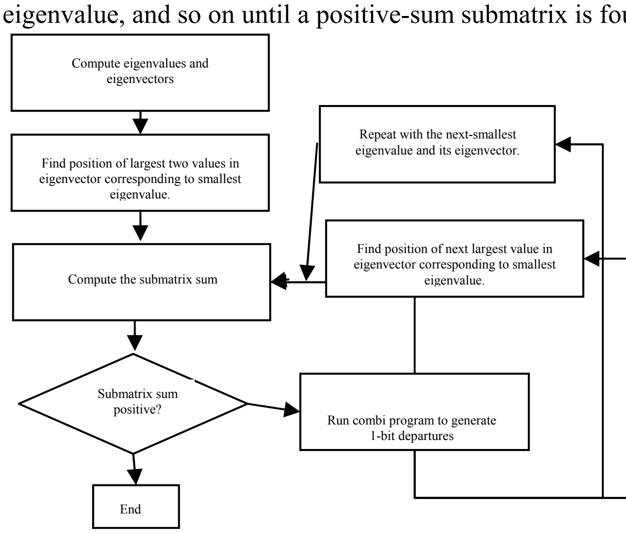

Fig. 2: Algorithm to find load pockets using sensitivity matrix eigenvalues

We compared the performance of the eigenvalue program with the earlier positive sum submatrix program, by running both programs 1000 times on the same 1000 randomly generated matrices and had the results shown in Table VII.

Table VII: Comparison of Submatrix and eigenvalue Submatrix Programs Performance

Program	Original Positive Sum Submatrix	Eigenvalue Based Positive Sum Submatrix	Eigenvalue Speed / Original Speed
Average time	5.9247e-7	2.6664e-7	2.22
Maximum time	6.8749e-6	1.2616e-6	5.45
Minimum time	1.851e-7	<1e-7	N/A

A. Heuristic method using eigenvalues and eigenvectors

The previously presented method using eigenvalues and eigenvectors to speed up the combinatorial search does not present significant improvements; on average it only halves the running time of the search program, so that it is still unacceptably slow for large systems. Instead of performing the combinatorial search for positive-sum submatrices, we now investigate using eigenvalues and eigenvectors as a heuristic to suggest groups of generators that may form load pockets.

The spectral analysis method presented in the work by Lesieutre, Thomas and Mount [5] can be applied as follows to identify potential load pockets in a group of generators:

1. Compute eigenvalues and eigenvectors of the revenue-offer or dispatch-offer sensitivity matrix
2. Sort the eigenvalues
3. Look at the eigenvector corresponding to the second-smallest eigenvalue
4. Check the submatrix sum of generators corresponding to the elements of the eigenvector greater than the mean, or the mean plus the standard deviation.

We have implemented this method, and it works for matrices obtained from power systems with known load pockets. Since this method relies on the matrix properties, it is difficult to test on randomly generated matrices. Changing the parameter in the fourth step above allows detection of different types of load pockets. A very high value would locate only very ideal load pockets. In the 30 bus, 6 generator system, using values between the mean and mean + ½ standard deviation identify that generators 5 and 6 are in a load pocket. The program continues to identify the load pocket in the dispatch-bid sensitivity matrix for higher values, but fails with the revenue-bid sensitivity matrix. In computing the revenue matrix from the dispatch matrix, the eigenvalues and eigenvectors change. We will investigate this further in the following section.

B. Some remarks about Fiedler vectors

A Fiedler vector in a positive semidefinite and singular matrix A(G), with smallest eigenvalue zero, is the eigenvector associated with the second smallest eigenvalue, also called a(G), or the algebraic connectivity of G [11]. Fiedler vectors have been applied to partition matrices by Pothen, Simon and Liou [12], and to partition power system networks by DeMarco and Wassner [13]. From the Lesieutre paper [5], we see that the dispatch – offer sensitivity matrix for a case with an ideal load pocket has two zero eigenvalues. The eigenvector corresponding to one zero eigenvalue is a vector of ones. The matrix M of dispatch-to-offer sensitivity is singular with a zero eigenvalue. However, it is not positive definite since it has zero and negative eigenvalues. Generating the dispatch-to-offer sensitivity matrix for several different power systems, we find that its eigenvalues are always zero and less, so that the eigenvalues of -1*M are always zero and greater. Thus, -1 times the matrix is positive semidefinite, and we may be able to apply some of the results relating to Fiedler vectors to this matrix.

Fiedler vectors are applicable to identifying load pockets: the eigenvector associated with the second zero eigenvalue (the Fiedler vector) can be divided into negative and positive elements. As seen in [5], if the load pocket is ideal, the negative elements will be identical, and a vector of generators in the load pocket can be determined. Running the two OPFs on a six generator, thirty bus system with a known load pocket, we found that there are two eigenvalues close to zero, and applying the above-described mathematics identifies a load pocket associated with the larger of the two.

We can implement this, adding a few steps to the heuristic method described earlier.

1. Compute eigenvalues and eigenvectors of M (the dispatch-bid sensitivity matrix)
2. Are there two nearly-zero eigenvalues?
3. Is one of the eigenvectors a vector of ones?
4. Look at the eigenvector associated with the other nearly-zero eigenvalue
5. Separate it into negative and positive elements
6. Use equation (24) from the Lesieutre paper, to get a vector whose nonzero elements are generators in a load pocket.

The first heuristic eigenvalue method relies on values in the eigenvector being above or below some parameter (the mean of the eigenvector, for example). We notice that this parameter can be higher for the dispatch matrix than in the revenue matrix with the program still locating the load pocket. This is because the eigenvalues and eigenvectors of the two matrices are not the same. While the dispatch matrix has the clear distinction of negative and positive values in the eigenvector corresponding to the second smallest eigenvalue in a system with a load pocket, the revenue matrix does not. When a load pocket is present, the revenue eigenvector does have some values much higher than others, but some values that would be negative in the dispatch matrix become small and positive in the revenue matrix. This method will work as long as the eigenvalues and eigenvectors of the matrix are computable. The Matlab function eig works until matrices are larger than 1000 x 1000. The function eigs may be faster since the user can specify how many of the smallest or largest eigenvalues and eigenvectors to calculate, but it does not work for matrices much larger than 1000 x 1000.

C. Using an augmented sparse matrix

One drawback in computing the sensitivity matrices as done in [5] is that for larger systems, it does not preserve the sparsity of the matrices, making eigenvalue calculations more time-consuming. The incremental model given in [5] is reduced to obtain the dispatch-to-offer sensitivity matrix. However, a similar result can be obtained by creating a large sparse matrix. We are currently working on extending this idea.

The matrix is constructed from the following equations developed in [5]:

$$0 = H\Delta y + \frac{\partial g_1(z,y)^T}{\partial y}\Delta\lambda_1 + \frac{\partial g_2(y)^T}{\partial y}\Delta\lambda_2$$

$$0 = -\Delta z + \frac{\partial g_1}{\partial y}\Delta y$$

$$0 = \frac{\partial g_2}{\partial y}\Delta y$$

Where H is the Hessian matrix of second derivatives from the OPF, g_1 is the binding active power constraints at generator buses, g_2 is the rest of the binding constraints, z is the dispatch, y is load constants, reactive powers, voltage magnitudes and angles, and λ_1 and λ_2 are the Lagrange multipliers for the constraints. The structure of the large sparse matrix is as follows in Table VIII.

For example, in the 30-bus, 6 generator system studied in [5], this matrix is 124 x 124, and is only 7.7% filled, in contrast to the 6 x 6 reduced matrix which is 100% filled. Fig. 3, Fig. 4 and Fig. 5 show the sparsity structure for a 118 bus, 54 generator system. In these figures, dots correspond to nonzero entries in the matrix. Fig. 3 shows the 54 x 54 reduced matrix, which is 74.1% filled; Fig. 4 illustrates the 499 x 499 augmented matrix, which is only 1.97% filled, and Fig. 5 shows a re-ordered version of the augmented matrix.

Table VIII: Augmented matrix structure

Δy	$\Delta\lambda_1$	$\Delta\lambda_2$	Δz
H	$\frac{\partial g_1^T}{\partial y}$	$\frac{\partial g_2^T}{\partial y}$	0
$\frac{\partial g_1}{\partial y}$	0	0	-I
$\frac{\partial g_2}{\partial y}$	0	0	0
0	-I	0	0

Table IX displays statistics measuring the sparsity of the reduced and augmented matrices. These statistics relate to the computational complexity of different computations with a given matrix [14]. τ is the total number of nonzero elements in the matrix, α is a measure of the computational effort to factor the matrix, and β is a measure of the computational effort to invert the matrix.

$$\alpha = \sum_{i=1}^{n} r_i * c_i$$

$$\beta = \sum_{i=1}^{n} (n-i) * r_i$$

Table IX: Sparsity indices for the 118 bus system reduced and augmented dispatch-offer sensitivity matrix

Matrix	size	% filled	τ	α	β
Reduced	54 x 54	71.6 %	2088	90146	61496
Augmented	499 x 499	1.97 %	4905	67193	1513607
Augmented Re-ordered	499 x 499	1.97 %	4905	45477	1100274

The augmented dispatch to offer sensitivity matrix has many of the same properties as the reduced matrix. It is symmetric, has all real eigenvalues and has at least one zero eigenvalue. However, its diagonal entries are all positive or zero and its eigenvalues are positive, negative, and zero. It may be possible to develop faster-running eigenvalue methods on the augmented matrix, and then apply a transformation to determine how the eigenvalues of the augmented and reduced matrices relate.

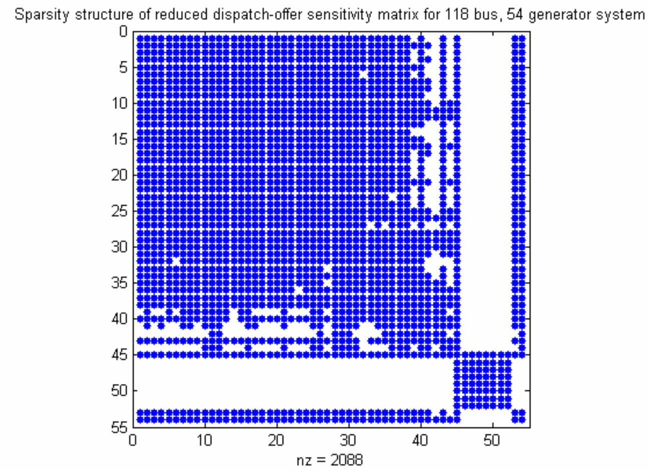

Fig. 3: Sparsity structure of reduced matrix for 118 bus, 54 generator system, 71.6% filled, τ = 2088, α = 90146, β = 61496

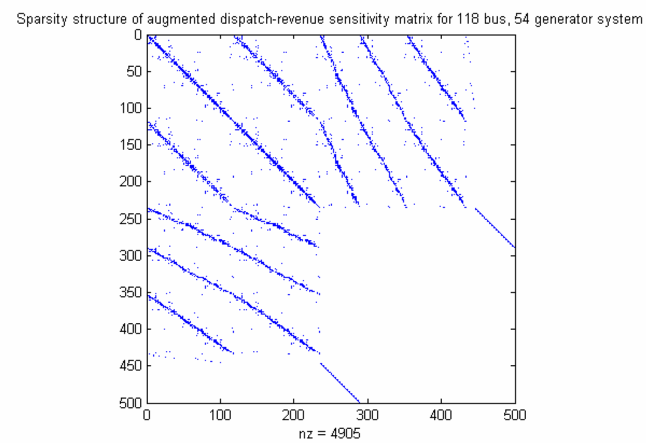

Fig. 4: Sparsity structure of augmented matrix for 118 bus system, 1.97% filled, τ = 4905, α = 67193, β = 1513607

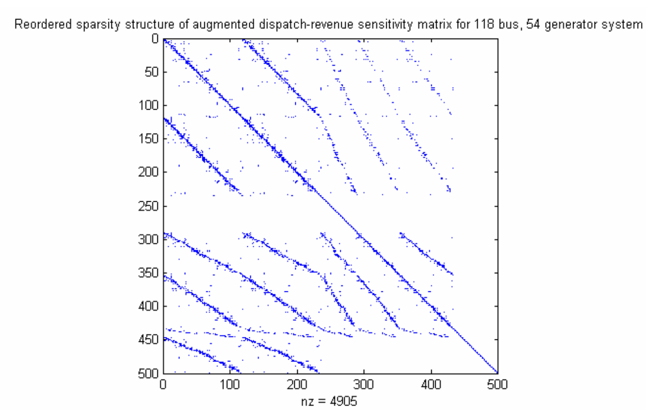

Fig. 5: Sparsity structure of re-ordered augmented matrix for 118 bus system, 1.97% filled, τ = 4905, α = 45477, β = 1100274

V. Conclusions

A matrix of dispatch or revenue to bid sensitivities can be a useful tool in locating load pockets and identifying groups that might benefit from market power. Some methods for identifying load pockets are: row sums, column sums, submatrix sums, and eigenvalues and eigenvectors. The easiest of these is to compute row sums, but many of these are false alarms – collusion benefits only one or a small part of the group, and decreases the revenue of the rest of the group. The positive-sum submatrix method is a more accurate metric for predicting market power potential. However, locating positive-sum submatrices in a large system is a computational challenge. Heuristic methods based on eigenvalues and eigenvectors will be more practical for larger systems. Furthermore, for very large systems, it may be more efficient to compute a large sparse matrix for the sensitivity instead of reducing the linearized equations into a small matrix, so that the needed eigenvalues and eigenvectors can be computed in a reasonable amount of time.

Future work in this area includes further investigation of the relationship between positive-sum subsets and eigenvectors and eigenvalues, time-domain analysis of the stability of collusive groups, and a similar analysis of the sensitivity of profit to bid. We are also investigating using a large sparse matrix based on the incremental model equations, looking at examples of exercise of market power by lowering bid, and analyzing how an ISO or FERC could incorporate the more detailed information available to them into similar methods. Historical bidding information and holding company arrangements could also be incorporated into this research.

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